**Purpose:** In this problem set, you will begin to explore exponential functions through real-world applications.

1. Suppose you need to grow bacteria in a petri dish to test out a new antibacterial medication. Which of the following tables models the population of the bacteria over time?

| Time (hours) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Population (million) | 10 | 20 | 40 | 80 | 160 |

| Time (hours) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Population (million) | 10 | 20 | 30 | 40 | 50 |

2. Suppose you are tracking the amount of radioactivity material in the water after a nuclear power plant spill. Which of the following tables models the amount of radioactive material over time?

| Time (years) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Amount (mg/L) | 10 | 9 | 8 | 7 | 6 |

| Time (years) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Amount (mg/L) | 10 | 5 | 2.5 | 1.25 | 0.625 |

3. Return back to the previous tables that you've selected. Can you write a mathematical formula for the data?

**Definition:** An **exponential function** is a function which grows or decays at a rate proportional to its current value. The formula looks like

$$f(x) =$$

We have a lot of questions about these functions and questions that these functions will help us answer.

- What are the domain and range of an exponential function?
- What does the graph of an exponential function look like?
- What is the inverse of an exponential function?
- How to make $1,000,000 dollars by the time you retire.

Some of these questions are easier to answer than others.

3. What is the domain of the exponential function $f(x) = b^x$ (where $b > 0$)?

4. What is the range of the exponential function $f(x) = b^x$ (where $b > 0$)?

5. Let's compare exponential functions to linear functions.

(a) On the same set of axes, graph $f(x) = 2^x$ and $g(x) = 2x$. Sketch the graph below and compare and contrast the graphs.

(b) Do the same but with $g(x) = 2x^2$. How does this quadratic compare with the exponential function?

(c) Do you think there's a polynomial that will grow faster than $f(x) = 2^x$? Graph them together (and sketch them below) and zoom out really far. What do you notice?

(d) FACT:

Let's talk **MONEY!**

6. Suppose you invest \$10 in an account which grows at 5% annually. How much money will you have after 3 years?

7. Suppose you invest \$0.93 in an account which grows at 2.25% annually. How much money will you have after 1,000 years?

8. After you graduate and take your first job, suppose you buy a car for \$32,000. If this car depreciates in value at 10% per year, what is the value of the car after 5 years?

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Suppose the annual interest rate a savings account gives is compounded monthly instead of annually. Then the interest rate will be different than percentage return on your investment at the end of the year.

**Definition:** The actual percent return on your investment at the end of the year is called **annual percentage yield** or APY.

Given an annual percentage rate of $r\%$ compounded $k$ times per year, the annual percentage yield is

$$\text{APY} = \left(1 + \frac{r}{k}\right)^k - 1$$

Interest could be compounded yearly, monthly, daily, hourly, minute-ly, or second-ly! If it's a unit of time, we can compound interest that often. What about *continuously*?

As $k \to \infty$, $\left(1 + \frac{r}{k}\right)^k \to$ ________________

This number is so common in math, economics, and elsewhere, we give it a name ... "$e$".

If we invest $\$P$ at $r\%$ APR which is compounded continuously for $t$ years, we will end up with ________________

9. How much money will you have in your account if you invest \$100 at 2% APR for 5 years, compounded continuously?

**Summary:**

- General Exponential Function:

$$y = ab^x + k, \text{ for } b > 0.$$

- Appreciation/Depreciation of $r\%$ per unit time $t$:

$$A = P(1 + r)^t.$$

- Appreciation/Depreciation of $r\%$ per unit time compounded continuously:

$$A = Pe^{rt}.$$

**For home:** How much money will you have to invest at 10% APR to end up with \$1,000,000 at the end of 40 years, compounded annually, monthly, daily, hourly, and continuously?